EFFECT OF THE FQUATION OF STATE ON THF DISPERSION OF DETONATION PRODUCTS

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This paper presents the results of a numerical solution of the problem concerning the dispersion of detonation products for various forms of the equation of state.

The most widely used equation describing the expansion of detonation products is the isentrope, suggested in [1]:

$$p = A\rho^k \tag{1}$$

where, for high explosives, $k \cong 3$.

However, Eq. (1) is valid only in the high-pressure region and does not make it possible to obtain the complete equation of state of the detonation products. This explains the appearance of the large amount of work devoted to studying the equation of state of explosion products (see, for example, [2, 3]).

In order to obtain the equation of state of detonation products, a specified functional relation between the parameters p, ρ, and E is usually assigned, the constant coefficients of which are found from experimental data or based on calculations.

The existence of different equations of state for the detonation products is a question of the effect of its form on the results of solving gas-dynamic problems. This problem is touched upon in [4], in which for one and the same explosive, the equation of state of the detonation products has been used in two forms.

In order to solve practical problems, it is desirable to choose the simplest form of the equation of state which will provide sufficient accuracy.

The representation of the isentrope of the detonation products in the form of a power–law two-term equation

$$p = A\rho^n + B\rho^{\gamma+1} \qquad (n > \gamma + 1) \tag{2}$$

is a further refinement of Eq. (1).

This dependence permits us to represent the isentrope in the form of a smooth curve (unlike [5]), which describes sufficiently well the expansion of the detonation products not only in the region of high pressures, but also of low pressures.

We obtain from Eq. (2) an expression for the logarithmic slope of the isentrope k

$$k = \frac{\rho}{p}\frac{dp}{d\rho} = n - B\frac{\rho^{\gamma+1}}{p}[n - (\gamma+1)] = n - B\frac{n-(\gamma+1)}{A\rho^{n-(\gamma+1)} + B} \tag{3}$$

whence it follows, that when $\rho \to 0$,

$$k \to k_0 = 1 + \gamma \tag{4}$$

For high explosives [5], k_0=1.25-1.35.

By applying the first law of thermodynamics to (2), the complete equation of state of the detonation products is obtained in the Mie–Grüneisen form

$$p = A[1 - \gamma/(n-1)]\rho^n + \gamma\rho E \tag{5}$$

This equation can be used for solving more complex adiabatic problems.

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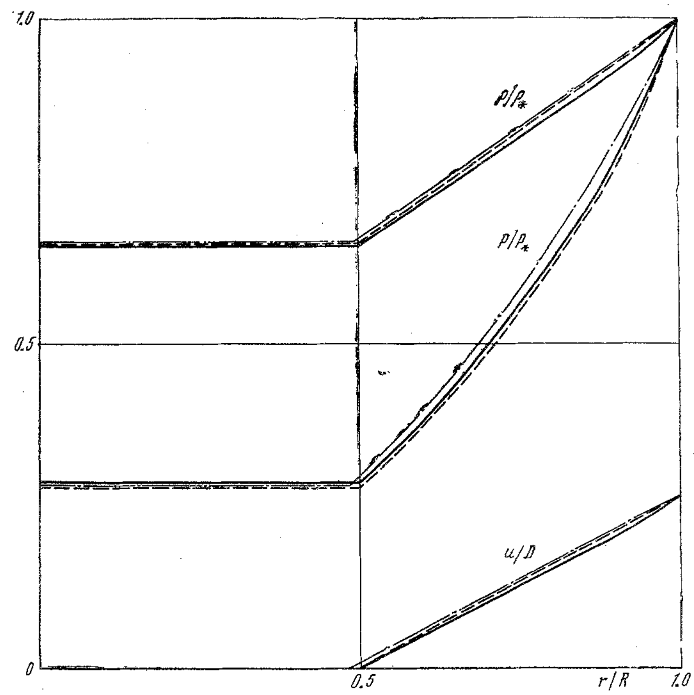


Fig. 1

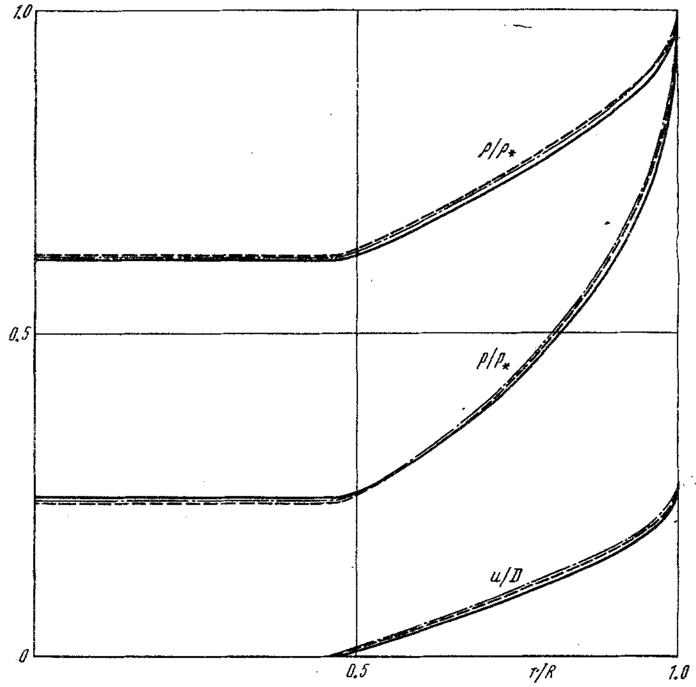


Fig. 2

The coefficients A, B, n and γ are determined by the parameters at the Chapman–Jouguet point

$$p_* = A\rho_*^{\,n} + B\rho_*^{\,\gamma+1} \tag{6}$$

$$k_* = n - B\rho_*^{\,\gamma+1}(n - \gamma - 1) / p_* \tag{7}$$

$$p_* = A[1 - \gamma / (n - 1)]\,\rho_*^{\,n} + \gamma\rho_* E_* \tag{8}$$

$$E_* = {}^1\!/_2\, p_* (1 / \rho_0 - 1 / \rho_*) + Q \tag{9}$$

where ρ_0 and Q are the density and heat of explosion of the explosive.

By using Eq. (4), the system of equations (6)–(9) completely defines the coefficients in the equation of state (5) and the isentrope (2).

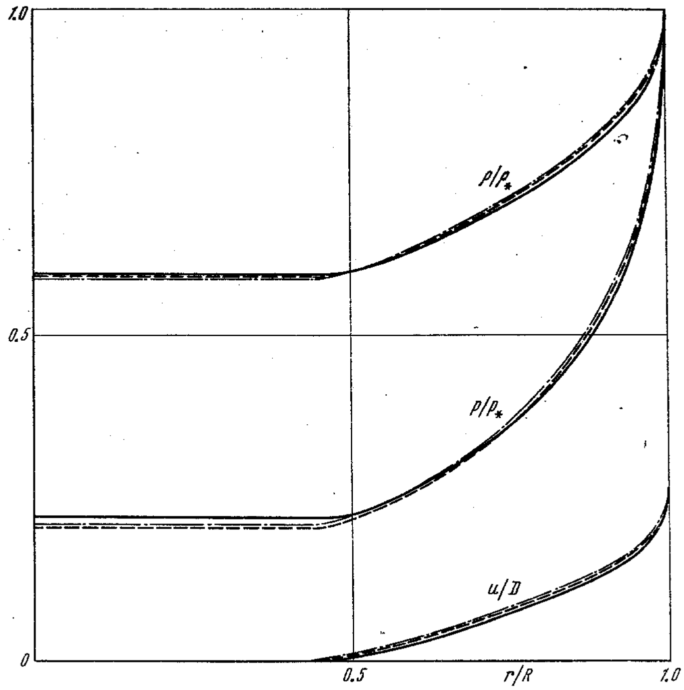


Fig. 3

In order to verify the feasibility of using the isentrope of the detonation products in the form of Eq. (2) for solving gas-dynamic problems for two explosives (Composition B and hexogen), problems concerned with plane, cylindrical and spherical detonations were solved numerically, in addition to the problem of dispersion of the detonation products in vacuo behind a plane detonation wave front. The isentropes obtained in [2, 3] were used together with Eq. (2) for the chosen explosives.

For Composition B [3]:

$$p = A\exp\left(-\frac{R_1\rho_0}{\rho}\right) + B\exp\left(-\frac{R_2\rho_0}{\rho}\right) + G\rho^{\gamma+1} \tag{10}$$

where the coefficients A, B, R_1, R_2, G and γ were taken from [3].

For hexogen, a power approximation was selected according to data from [2] for the index k:

$$k = \frac{\rho}{p}\frac{dp}{d\rho} = a_0 + a_1\rho + a_2\rho^2 + a_3\rho^3 + a_4\rho^4 + a_5\rho^5 \tag{11}$$

which, then integrated for the relief isentrope, gives

$$p = G\rho^{a_0}\exp\left(a_1\rho + a_2\rho^2/2 + a_3\rho^3/3 + a_4\rho^4/4 + a_5\rho^5/5\right) \tag{12}$$

Relation (1) was used together with Eq. (2), (10) and (12) for solving the problems.

Figures 1, 2 and 3 show the pressure distribution p, density ρ, and mass velocity u, respectively, for plane, cylindrical and spherical detonation waves in hexogen. The solid lines refer to the results obtained for Eq. (12); the dashed lines are for Eq. (1) and the chained lines are for Eq. (2). Similar results are obtained for Composition B.

It can be seen from the figures that, in the case of detonation for any symmetry, all three isentropes give almost coincident results. (It must be taken into account that in the figures all solutions are reduced to the parameters at the detonation front, which correspond to isentrope (12). Therefore, for Eq. (1) $u/D \neq 0.25$ at the front.) Because of this, for practical calculations, obviously there is no point in giving preference to more complex isentropes in comparison with Eq. (1) in these problems, especially as in the case where Eq. (1) is used, the solution obtained is suitable for any explosive.

The results of the solution to the problems concerning cylindrical and spherical detonation waves are approximated, with a high degree of accuracy, by the relations

$$p/p_* = 1 - A_p(R/r - 1)^{0.5} + B_p\left[(R/r)^{\beta_p} - 1\right]$$

$$u/D = 0.25 - A_u(R/r - 1)^{0.5} + B_u\left[(R/r)^{\beta_u} - 1\right], \text{ where } r_1 \leqslant r \leqslant R \tag{13}$$

$$p/p_* = p_1, \quad u/D = 0 \quad \text{where } 0 \leqslant r \leqslant r_1 \tag{14}$$

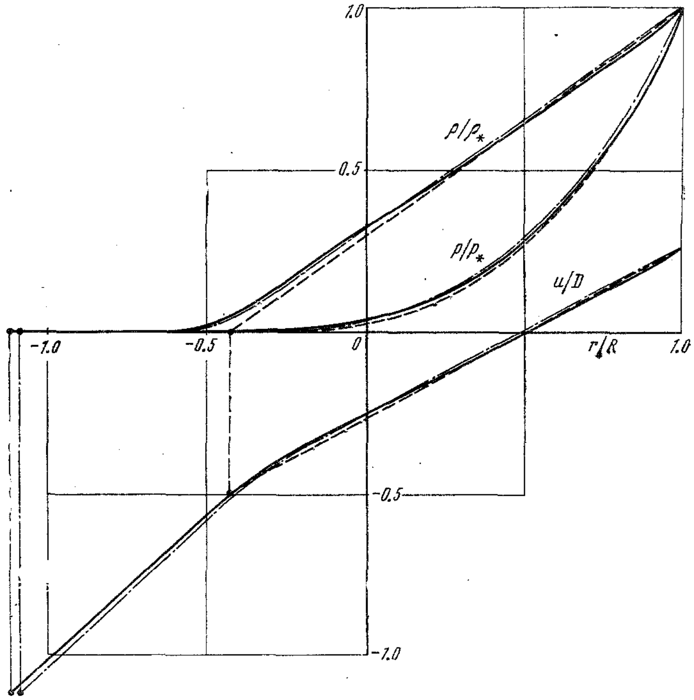
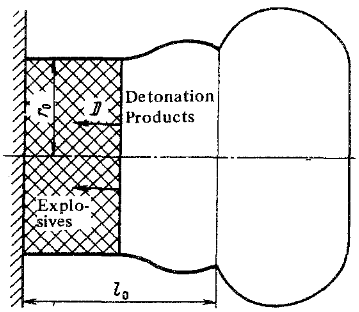


Fig. 4



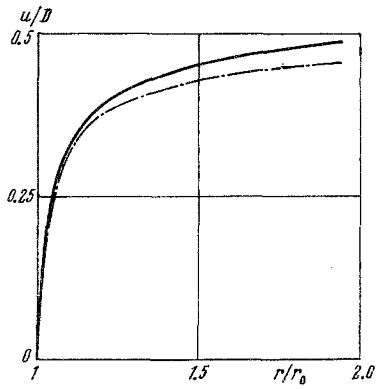


Fig. 5 Fig. 6

where R is the radius of the detonation front; r_1 is the radius of the steady-state zone, and the coefficients in Eq. (13) and (14) are equal to

for a spherical wave:

$$\begin{array}{llll} A_p = 1.73, & B_p = 3.103, & \beta_p = 0.39, & p_1 = 0.223 \\ A_u = 0.421, & B_u = 0.297, & \beta_u = 0.68, & r_1 = 0.455R \end{array} \tag{15}$$

for a cylindrical wave:

$$\begin{array}{llll} A_p = 1.294, & B_p = 0.703, & \beta_p = 0.84, & p_1 = 0.249 \\ A_u = 0.3002, & B_u = 0.0163, & \beta_u = 2.18, & r_1 = 0.472R \end{array} \tag{16}$$

In problems where expansion of the detonation products takes place down to pressures close to zero, the use of Eq. (1) leads to considerable errors. This can be seen from Fig. 4, which shows the results of solving the problem of dispersion of detonation products in vacuo behind the front of a plane detonation wave in hexogen (the notation in Fig. 4 is the same as in the previous figures). At the same time, isentropes (2) and (12) give almost coincident results.

In order to study the feasibility of using the equation of state of the detonation products in the form of Eq. (5) in adiabatic problems, a variant of the problem in [6] was claculated; this concerned the disintegration of a cylindrical shell (with load factor $\beta = 2$ and linear expansion $l_0/r_0 = 2$) under the action of a glancing detonation wave. A diagram of the process is shown in Fig. 5.

This problem was solved for pentolite both by means of Eq. (5) and also by using the equation of state for the detonation products

$$p = A\rho E + B\rho^4 + G \exp(-k/\rho) \tag{17}$$

which was derived in [4].

Figure 6 shows graphs of a set of shell velocities close to a rigid wall for both equations [the solid line corresponds to Eq. (17) and the chained line corresponds to Eq. (5)]. The difference in the final results is $\sim 6\%$.

The analysis carried out shows that, for calculations of gas-dynamic problems of the effect of an explosion, the simple equation of state (5) and its isentrope (2) can be used with a sufficient degree of accuracy; the coefficients in these equations are determined for any explosive by the known parameters at the Chapman–Jouguet point.

The existence of the isentrope of the detonation products in the analytical form of Eq. (2) simplifies considerably the solution of isentropic problems.

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